Exercice n°1(3,5 points) : $z_1 = \sqrt{3} - 3i$, $z_2 = -2+i\sqrt{3}$

$\overline{z_1}.z_2 = (\sqrt{3}+3i)(-2+i\sqrt{3}) = -2\sqrt{3} + i\sqrt{3}\times\sqrt{3} - 6i + 3\sqrt{3}i^2 = -2\sqrt{3} + 3i - 6i - 3\sqrt{3} = -5\sqrt{3} - 3i$.

$(z_1)^2 = (\sqrt{3} - 3i)^2$

On utilise l'identité remarquable **$(a - b)^2 = a^2 - 2ab + b^2$** avec ici $a = \sqrt{3}$ et $b = 3i$

D'où $(z_1)^2 = \sqrt{3}^2 - 2\sqrt{3}\times 3i + (3i)^2 = 3 - 6\sqrt{3}i + 9i^2 = 3 - 6\sqrt{3}i - 9 = -6 - 6\sqrt{3}i$,

$\dfrac{1}{z_1} = \dfrac{1}{\sqrt{3}-3i} = \dfrac{\sqrt{3}+3i}{(\sqrt{3}-3i)(\sqrt{3}+3i)} = \dfrac{\sqrt{3}+3i}{\sqrt{3}^2+(3)^2} = \dfrac{\sqrt{3}+3i}{3+9} = \dfrac{\sqrt{3}}{12} + \dfrac{1}{4}i$,

Attention : $(\sqrt{3}-3i)(\sqrt{3}+3i)$ est égal à $(\sqrt{3})^2 - (3i)^2$ ou à $(\sqrt{3})^2 + (3)^2$ mais pas à $(\sqrt{3})^2 + (3i)^2$
Rappel : $(a-b)(a+b) = \mathbf{a^2 - b^2}$

$\dfrac{z_2}{z_1} = \dfrac{-2+i\sqrt{3}}{\sqrt{3}-3i} = \dfrac{(-2+i\sqrt{3})(\sqrt{3}+3i)}{3+9} = \dfrac{-2\sqrt{3}-6i+3i-3\sqrt{3}}{12} = \dfrac{-5\sqrt{3}-3i}{12} = \dfrac{-5\sqrt{3}}{12} - \dfrac{1}{4}i$

Exercice n°2 (4,5 points) :

1. $\dfrac{2z+1}{z-3} = -1+i \Leftrightarrow 2z+1 = (-1+i)(z-3) \Leftrightarrow 2z+1 = -z+3+iz-3i \Leftrightarrow 3z - iz = 2-3i \Leftrightarrow (3-i)z = 2-3i$

$\Leftrightarrow z = \dfrac{2-3i}{3-i} \Leftrightarrow z = \dfrac{(2-3i)(3+i)}{(3)^2+(-1)^2} \Leftrightarrow z = \dfrac{6+2i-9i+3}{9+1} \Leftrightarrow z = \dfrac{9-7i}{10}$

2. $\begin{cases} z - 2z' = -2+3i \\ z - iz' = 4-2i \end{cases} \Leftrightarrow \begin{cases} z - 2z' = -2+3i \\ -z + iz' = -4+2i \end{cases}$

En additionnant membre à membre on obtient : $-2z'+iz'=-6+5i$

$z'(-2+i) = -6+5i \Leftrightarrow z' = \dfrac{-6+5i}{-2+i} \Leftrightarrow z' = \dfrac{(-6+5i)(-2-i)}{4+1} \Leftrightarrow z' = \dfrac{12+6i-10i+5}{5} \Leftrightarrow z' = \dfrac{17-4i}{5}$

$z - 2z' = -2+3i$ donc $z = -2+3i+2z' = -2+3i+2\left(\dfrac{17}{5} - \dfrac{4}{5}i\right) = -2+3i+\dfrac{34}{5} - \dfrac{8}{5}i = \dfrac{24}{5} + \dfrac{7}{5}i$.

Conclusion : le système a pour solution $z = \dfrac{24}{5} + \dfrac{7}{5}i$ et $z' = \dfrac{17-4i}{5}$

Exercice n°3 (2 points) :

Rappel : $\dfrac{a}{b}\times\dfrac{c}{d} = \dfrac{ac}{bd}$ donc $\dfrac{\sqrt{3}}{2}\times\dfrac{\sqrt{3}}{2} = \dfrac{\sqrt{3}\times\sqrt{3}}{2\times 2} = \dfrac{(\sqrt{3})^2}{4} = \dfrac{3}{4}$

$$\left(\frac{\sqrt{2}}{3} - \frac{2\sqrt{3}}{2}\right)\left(\frac{\sqrt{3}}{2} - \frac{3\sqrt{2}}{2}\right) = \frac{\sqrt{2}\times\sqrt{3}}{3\times 2} - \frac{\sqrt{2}\times 3\sqrt{2}}{3\times 2} - \frac{2\sqrt{3}\times\sqrt{3}}{2\times 2} + \frac{2\sqrt{3}\times 3\sqrt{2}}{2\times 2} = \frac{\sqrt{6}}{6} - \frac{3\times 2}{6} - \frac{2\times 3}{4} + \frac{6\times\sqrt{6}}{2\times 2} = \frac{\sqrt{6}}{6} - \frac{6}{6} - \frac{6}{4} + \frac{3\times\sqrt{6}}{2}$$

$$= \frac{\sqrt{6}}{6} - \frac{12}{12} - \frac{18}{12} + \frac{9\times\sqrt{6}}{6} = \frac{10\sqrt{6}}{6} - \frac{30}{12}$$

$$= \frac{5\sqrt{6}}{3} - \frac{5}{2}$$